Fermat's factorization method of N=a^2 - b^2 geometrically can be viewed as the Pythagorean theorem, in fact Fermat's factorization method generates Pythagorean Triples when N is a perfect square. Expanding on this geometry:

Given a divisor *k*, a dividend *n*, and a quotient *q* , a symmetry can be obtained by comparing the divisor and quotient. With the difference, t between the quotient and divisor

$$t = q - k$$

The polynomial equation

$$kq - n = 0$$

$$k(k + t) - n = 0$$

is solved by

$$t = (n - k^2)/k.$$

$D(n) = \sum_{k=1}^{\lfloor\sqrt{n}\rfloor}(2 \cdot \lfloor t \rfloor + 1)$ (The Divisor Summatory Function)

$\frac{t}{2}$ provides some useful geometric insite.

$$s = \frac{t}{2}$$

alternate forms:

$$s = (n - k^2)/(2k)$$

$$s + k = (n + k^2)/(2k)$$

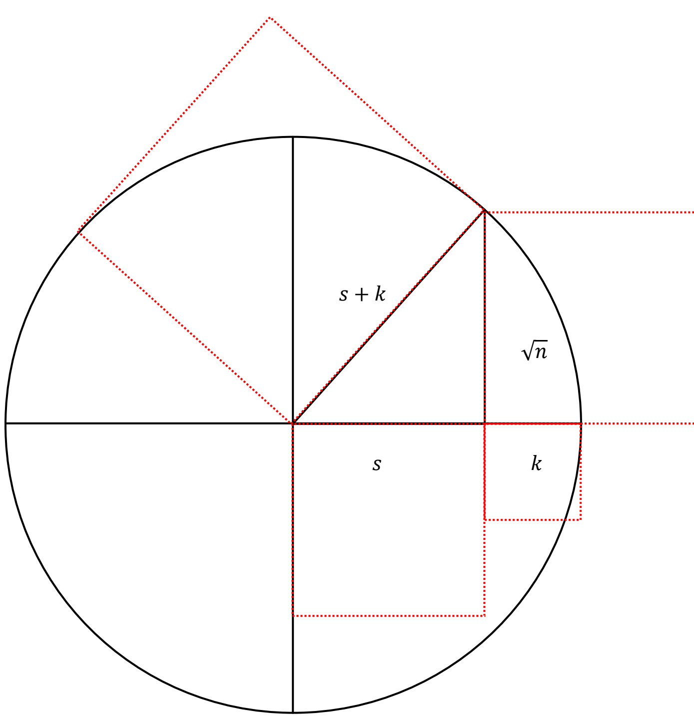
$(s + k)^2 - s^2 = n$ (Fermat's factorization method N=a^2 - b^2)

$$\sqrt{(s + k)^2 - n} = s$$

$$(s + k) + s = q$$

$$(s + k) - s = k$$

$$(s + k) + \sqrt{(s + k)^2 - n} = \frac{n}{k} = q$$



Klein four-group of the function $= (n - k^2)/(2k)$.

Z/Z_12	0	1	2	3	4	5	6	7	8	9	10	11	Additive group of integers modulo m , where m=12
QR_12*	0	1	4	9	4	1	0	1	4	9	4	1	Quadratic residue of the multiplicative group modulo m
Z_12*		1				5		7				11	Multiplicative group of integers modulo m, Klein four-group
s^2		0				4		9				1	(s+k)^2-s^2=Z_12*, permutation of QR_12*={0,4,9,1}
(s+k)^2		1				9		4				0	(s+k)^2-s^2= Z_12*, permutation of QR_12*={1,9,4,0}
s		{0,6}				{2,8}		{3,9}				{5,11}	{0,2,3,5,6,8,9,11}
s+k		{7,1}				{9,3}		{10,4}				{0,6}	{1,3,4,6,7,9,10,0}+{0,2,3,5,6,8,9,11}={1,5,7,11,13,17,19,23}
k		{7,5}				{7,5}		{7,5}				{7,5}	{7,5} a generating set of Z12* Klein four-group

example n=119, m=12

119 = 11 (mod 12)

$s = \frac{119-1}{2} = 59, \quad \mathrm{s}^2 = 1\ (\mathrm{mod})12$

$s = \frac{119-49}{14} = 5, \quad \mathrm{s}^2 = 1\ (\mathrm{mod})12$

$s = \frac{119-289}{34} = -5, \quad \mathrm{s}^2 = 1\ (\mathrm{mod})12$

$s = \frac{119-14161}{238} = -59, \quad \mathrm{s}^2 = 1\ (\mathrm{mod})12$

$s + k = \frac{119+1}{2} = 60, \quad (\mathrm{s}+\mathrm{k})^2 = 0\ (\mathrm{mod})12$

$s + k = \frac{119+49}{14} = 12, \quad (\mathrm{s}+\mathrm{k})^2 = 0\ (\mathrm{mod})12$

$s + k = \frac{119+289}{34} = 12, \quad (\mathrm{s}+\mathrm{k})^2 = 0\ (\mathrm{mod})12$

$s + k = \frac{119+14161}{238} = 60, \quad (\mathrm{s}+\mathrm{k})^2 = 0\ (\mathrm{mod})12$

Z/Z8	0	1	2	3	4	5	6	7	Group of integers in the additive group modulo 8
QR8*	0	1	4	1	0	1	4	1	Quadratic residue of the multiplicative group modulo 8
Z8*		1		3		5		7	Multiplicative group modulo 8, Klein four-group
S^2		0		1		4		1	(S+K)^2-S^2= Z8* permutation of QR8*={0,1,4,1}
(S+K)^2		1		4		1		0	(S+K)^2-S^2= M8 permutation of QR*8={1,4,1,0}
S		{0,4}		{1,5}		{2,6}		{3,7}	{0,1,2,3,4,5,6,7}
S+K		{1,5}		{0,4}		{7,3}		{6,2}	{0,1,2,3,4,5,6,7}
K		{1,1}		{7,7}		{5,5}		{3,3}	{7,3},{1,5} generating sets of Z8* Klein four-group

example n=119, m=8

119 = 7 (mod 8)

$s = \frac{119-1}{2} = 59, \quad s^2 = 1 \text{ (mod)}8$

$s = \frac{119-49}{14} = 5, \quad s^2 = 1 \text{ (mod)}8$

$s = \frac{119-289}{34} = -5, \quad s^2 = 1 \text{ (mod)}8$

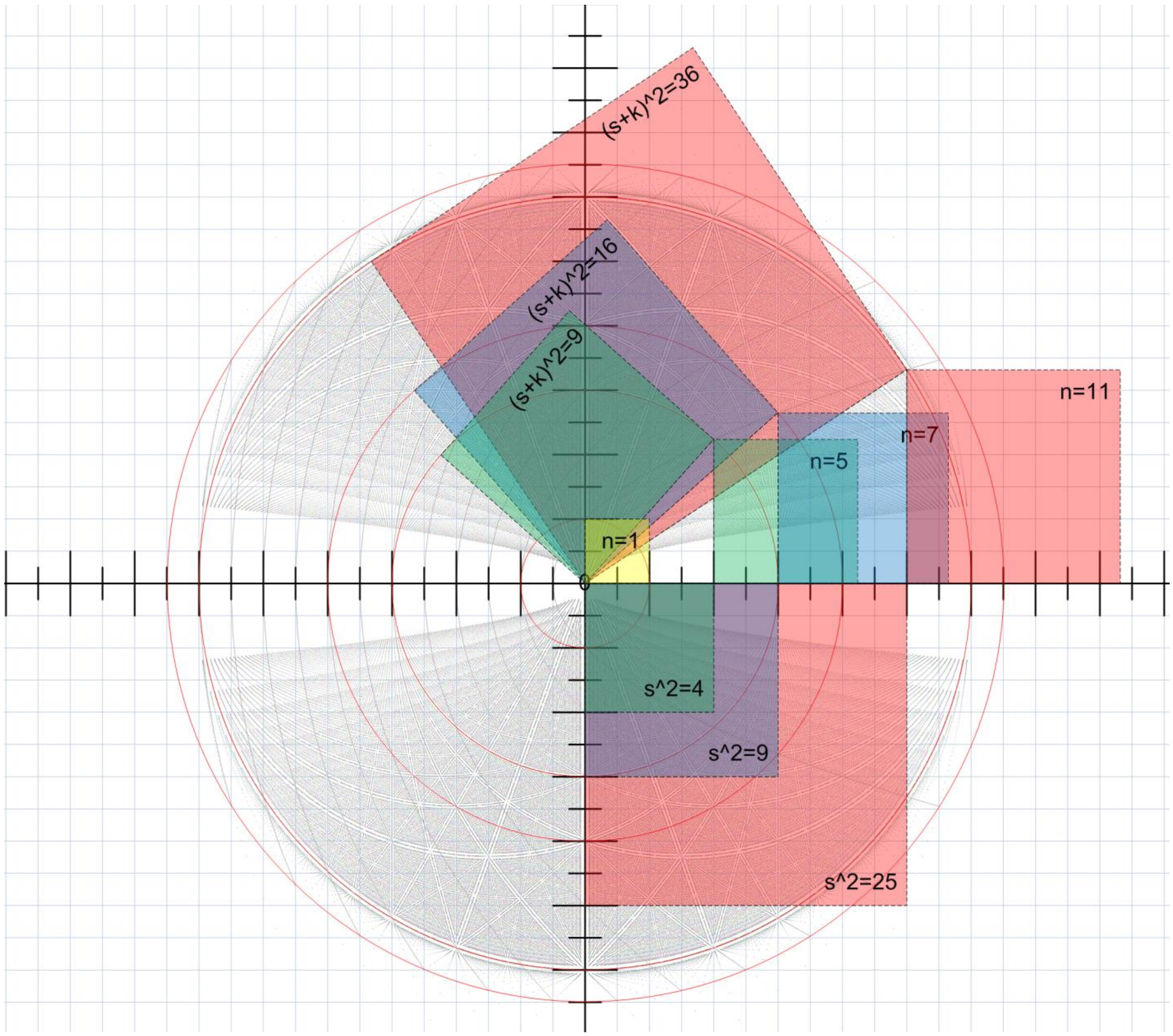
$s = \frac{119-14161}{238} = -59, \quad s^2 = 1 \text{ (mod)}8$

$s + k = \frac{119+1}{2} = 60, \quad (s+k)^2 = 0 \text{ (mod)}8$

$s + k = \frac{119+49}{14} = 12, \quad (s+k)^2 = 0 \text{ (mod) } 8$

$s + k = \frac{119+289}{34} = 12, \quad (s+k)^2 = 0 \text{ (mod) } 8$

$s + k = \frac{119+14161}{238} = 60, \quad (s+k)^2 = 0 \text{ (mod) } 8$


(s+k)^2=36
(s+k)^2=16
(s+k)^2=9
n=11
n=7
n=5
n=1
0
s^2=4
s^2=9
s^2=25

NOTES

The function $f: Q_n \rightarrow Q_n$ defined by $f(x) = x^2 \bmod n$ is a permutation. The inverse function of f is: $f^{-1}(x) = x^{((p-1)(q-1)+4)/8} \bmod n$

reduced residue system modulo 12 = {1,5,7,11} = n = $\varphi(12)$

quadratic residue modulo 12= {1,4,9,4,1,0} = {0,4,9,1} = s^2

quadratic residue modulo 12= {1,4,9,4,1,0} = {1,9,4,0} = (s+k)^2

1-0=1

9-4=5

4-9=7

0-1=11

least/complete residue system modulo 12 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

reduced residue system modulo 12 = {1,5,7,11} $\varphi(12) = 4,\ \lambda(12) = 2\ , totient\ and\ carmichael\ functions$

multiplicative group of integers modulo 12= $(\mathbb{Z}/12\mathbb{Z})^x \cong C_2 \times C_2,$ the Klein four-group

*	**1**	***a***	***b***	***ab***
1	1	*a*	*b*	*ab*
a	*a*	1	*ab*	*b*
b	*b*	*ab*	1	*a*
ab	*ab*	*b*	*a*	1

{1,12,66,220,495,792,924,792,495,220,66,12,1} = n!/(k!(n-k)!)

http://mathworld.wolfram.com/FiniteGroupC2xC2.html

$where\ 0 \leq x \leq \infty, \qquad x \in \boldsymbol{Z}$

$\boldsymbol{A} = 12x + 1$

$\boldsymbol{B} = 12x + 5$

$\boldsymbol{C} = 12x + 7$

$\boldsymbol{D} = 12x + 11$

Again looking at the geometries of the system from a quadratic residue mod 12 viewpoint:

$where\ s = (n - k^2)/(2k)$

$(s + k)^2 \ \equiv 1\ mod\ 12, \qquad s^2 \equiv 0\ mod\ 12, \qquad n \in \boldsymbol{A}$

$(s + k)^2 \ \equiv 9\ mod\ 12, \qquad s^2 \equiv 4\ mod\ 12, \qquad n \in \boldsymbol{B}$

$(s + k)^2 \ \equiv 4\ mod\ 12, \qquad s^2 \equiv 9\ mod\ 12, \qquad n \in \boldsymbol{C}$

$(s + k)^2 \ \equiv 0\ mod\ 12, \qquad s^2 \equiv 1\ mod\ 12, \qquad n \in \boldsymbol{D}$

bijection or one-to-one correspodence is a function giving an *exact* pairing of the elements of two sets

$\boldsymbol{A} + \boldsymbol{D} = \boldsymbol{A}, \qquad \boldsymbol{B} + \boldsymbol{C} = \boldsymbol{A}$

$\boldsymbol{A} - \boldsymbol{D} = \boldsymbol{A}, \qquad \boldsymbol{B} + \boldsymbol{C} = \boldsymbol{A}$

$\boldsymbol{D} \ - \boldsymbol{A} = \boldsymbol{C}, \qquad \boldsymbol{B} \cdot \boldsymbol{D} = \boldsymbol{C},$

$\boldsymbol{A} \cdot \boldsymbol{D} = \boldsymbol{D}, \qquad \boldsymbol{B} \cdot \boldsymbol{C} = \boldsymbol{D},$

$\boldsymbol{A} \cdot \boldsymbol{A} = \boldsymbol{A}, \qquad \boldsymbol{B} \cdot \boldsymbol{B} = \boldsymbol{A}, \qquad \boldsymbol{C} \cdot \boldsymbol{C} = \boldsymbol{A}, \qquad \boldsymbol{D} \cdot \boldsymbol{D} = \boldsymbol{A}$

$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B}, \qquad \boldsymbol{C} \cdot \boldsymbol{D} = \boldsymbol{B},$

$\boldsymbol{A} \cdot \boldsymbol{C} = \boldsymbol{C}, \qquad \boldsymbol{B} \cdot \boldsymbol{D} = \boldsymbol{C},$

$\boldsymbol{A} \cdot \boldsymbol{D} = \boldsymbol{D}, \qquad \boldsymbol{B} \cdot \boldsymbol{C} = \boldsymbol{D},$

$where\ 1 \le p \le (x + 1)$

$s + k \equiv \{1{,}5\} mod\ 6, \qquad s \equiv 0\ mod\ 6, \qquad n \in \boldsymbol{A}, \qquad s + k = \ \tfrac{1}{2}(6p + 3 - (-1)^p)$

$s + k \equiv 3\ mod\ 6, \qquad s \equiv \{2{,}4\} mod\ 6, \qquad n \in \boldsymbol{B}, \qquad s + k = \ (6p + 3)$

$s + k \equiv \{2{,}4\} mod\ 6, \qquad s \equiv 3\ mod\ 6, \qquad n \in \boldsymbol{C}, \qquad s + k = \ \tfrac{1}{2}(6p + 3 + (-1)^p)$

$s + k \equiv 0\ mod\ 6, \qquad s \equiv \{1{,}5\} mod\ 6, \qquad n \in \boldsymbol{D}, \qquad s + k = \ 6p$

A composite number example:

$n \in \boldsymbol{D}, \qquad n = 119, \qquad x = 9, \qquad p = \dfrac{n + k^2}{12k}, \qquad k = s + k - \ \sqrt{(s + k)^2 - n}, \quad q = s + k + \ \sqrt{(s + k)^2 - n}$

$\dfrac{119 - 1^2}{2} = 59, \qquad 59^2 \equiv 1\ mod\ 12, \qquad \{k, q\} = (6 * 10) \pm \sqrt{\ 6^2 * 10^2 - 119} = \{1{,}119\}$

$\dfrac{119 - 7^2}{14} = 5, \qquad 5^2 \equiv 1\ mod\ 12, \qquad \{k, q\} = (6 * 2) \pm \ \sqrt{\ 6^2 * 2^2 - 119} = \{7{,}17\}$

$\dfrac{119 - 17^2}{26} = -5, \qquad -5^2 \equiv 1\ mod\ 12, \qquad \{k, q\} = (6 * 2) \pm \ \sqrt{\ 6^2 * 2^2 - 119} = \{17{,}7\}$

$\dfrac{119 - 119^2}{238} = -59, \qquad -59^2 \equiv 1\ mod\ 12, \qquad \{k, q\} = (6 * 10) \pm \ \sqrt{\ 6^2 * 10^2 - 119} = \{119{,}1\}$

$$(n+k) \pm \sqrt{6^2 * 10^2 - 119} = \{119,1\}$$

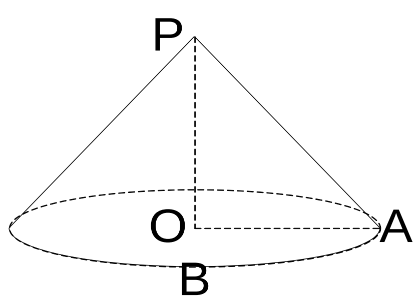
$$m\frac{n+k^2}{m2k} + \sqrt{m^2\left(\frac{n+k^2}{m2k}\right)^2 - n} = (2S+k)\ mod\ (2m)$$

s((n+k^2)/(2s*k)) + sqrt(s^2((n+k^2)/(2s*k))^2 - n)

$$\frac{k}{s+k} = versine(\theta)$$

$$\frac{s}{s+k} = cosine(\theta)$$

$$\frac{\sqrt{n}}{s+k} = sine(\theta)$$



B=2sqrt(n)

H=(n-k^2)/(2k)

The lateral surface area of a right circular cone is $LSA = \pi r l$ where r is the radius of the circle at the bottom of the cone and l is the lateral height of the cone (given by the Pythagorean theorem $l = \sqrt{r^2 + h^2}$ where h is the height of the cone). The surface area of the bottom circle of a cone is the same as for any circle, πr^2. Thus the total surface area of a right circular cone is:

$$SA = \pi r^2 + \pi r l$$ or

$$SA = \pi r(r + l)$$

[edit] Volume

See also: Pyramid (geometry)#Volume

The volume V of any conic solid is one third of the product of the area B of the base and the height H (the perpendicular distance from the base to the apex).

$$V = \frac{1}{3}BH$$

In modern math, this formula can easily be computed using calculus – it is, up to scaling, the integral $\int x^2 dx = \tfrac{1}{3}x^3$.